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Application of Stochastic Models for Capital Market Investments: Additive Effects Analysis

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Abstract
Accurate estimation of asset value changes in capital markets is fundamental to informed investment decision-making, particularly in highly volatile sectors. While stochastic models such as Geometric Brownian Motion (GBM), Jump Diffusion, and stochastic volatility models have been widely applied to capture price dynamics, these approaches do not incorporate time-dependent delay mechanisms that reflect realistic investor behaviour such as delayed asset liquidation in anticipation of price appreciation. The systems of stochastic equations were solved adopting Ito's theorem where explicit solutions for assessing asset values were obtained. Simulations were performed for varying levels of delay parameters and return rates (1.0000 and 2.0000), under constant volatility, to determine whether delayed strategies generate statistically superior financial outcomes in comparison to immediate asset liquidation. Most existing studies model price dynamics using geometric Brownian motion or stochastic volatility models without incorporating delay-dependent mechanisms. The analysis further revealed that an increase in return rates amplifies these gains, confirming that delay and rate of return interaction contribute positively to wealth accumulation. Goodness-of-fit tests using the Kolmogorov–Smirnov (KS) procedure showed that asset values with delay and without delay do not originate from the same distribution at a 1% significance level. This statistical divergence suggests that delay is not only financially advantageous but also structurally alters the stochastic behaviour of asset values, implying higher risk–return sensitivity. Moreover, graphical results corroborated these findings by displaying upward trajectories for delayed assets, contrasted with low or stagnant returns for non-delayed strategies. To this end, the study provided actionable insights for investors, portfolio managers, and policymakers seeking optimal timing strategies to enhance wealth creation in capital markets.

Keywords: *Stochatic Differential Equations, Stock Prices, Delay Parameter, Additive Effects and Investments*

Introduction

Generally, stochastic models have proven useful in capturing the complexities of asset price movements, making them essential in the estimation of asset value changes within capital markets. Investing on assets resulting in the using the returns to take care of the daily expenditures in capital assets. To improve on trading activities, an investor may need more capital investments. Thus, capital assets are put in to acquire more capital investments for growth which enables the investment to bring more yields in the unit production, produce new concepts on products or even introduce values to the business and to check modern technology in order to improve productivity and reduce cost and to replace the worn-out assets. Trading investments, where capital assets are absent, will absolutely have challenging time getting off the ground. The results of stock market undertaking and performances have raised considerably in the growth of financial market as investors gain their normal returns to have a living. Notwithstanding, stock business is usually known for its high returns. A return on investments for stock market is a medium of correctly estimating cash flow of an investment. It is the major parameter consistently applied by investors to handle the expenditure (Merton,2017). The dynamics of stock prices in the stock market are reproduced by unpredictable flow or movement of their value over time. For this reason, it is better to have an apt comprehension of the type of physical quantities to model so as to have pragmatic outcomes of the stock variables. The investment of assets in businesses creates a room where the returns or profits would be used for the management of the daily cost of the running businesses in financial investments. So, an investor might potentially require more capital investment so as to bring about the improvement of the investment proceedings. Accordingly, financial resources are invested in essence to obtain more asset investments for the purpose of expansion and growth of the business. This creates a room for improved unit productivities, creation of innovativeness for new products or bring in values to the investments and to look for modern technology that improves on productivity and minimizes cost and replace outdated assets. If capital assets are skipped, stock exchange will definitely be confronted with the challenges of getting off the ground. There has been progressive growth of stock market activities and performances in the financial market as investors earn incomes to have a living.

However, a lot of scholars has written extensively such as, Loko et al. (2023) studied the effect of Fourier series expansion on the solution of Stochastic Differential Equation (SDE). Okpoye et al. (2023) worked on an empirical assessment of asset value function for capital market price changes. While, Amadi and Wobo (2022) developed mathematical model analysis for estimating Stock Market Price Changes. Amadi and Okpoye (2022) considered the application of stochastic model in estimation of stock return rates in capital market investments. In a similar research work by Amadi and Vivian (2022) on: A stochastic analysis of stock price variation assessments in Oando Nigeria Plc, the authors employed stochastic analysis of Markov chain in the closing stock price data of Oando Nigeria Plc. In the same way, Amadi and Charles (2022) attempted to determine the impacts multiplicative and multiplicative inverse trends series have on the value of asset prices of investors in capital market. In another study by Amadi et al. (2022), Examined Stochastic analysis of Markov chain. Their findings reveales that the stock (S1) has a higher mean rate of return but Stock (S2) has the best chances of price increasing in the near future hence informing the investors to have good and proper idea and of the corresponding behavior of stocks as regards decision making. Osu et al. (2022) Studied Solutions to Stochastic boundary value problems in Sobolev Spaces

This paper considered two stochastic systems such as Stochastic Differential Equation (SDE) and Stochastic Delay Differential Equation (SDDE) respectively which gave précised conditions for determining asset values through additive effects series. The validity of the analytical solution was verified using initial stock prices. Hence, this paper extends the works of Amadi and Okpoye (2022) by incorporating time dependent delay parameter to realistically assess asset pricing function. The advantage of this study over the works of Amadi and Okpoye (2022) and Okpoye et, al., (2023) is that the present work models the impact of Stochastic Delay Differential Equation (SDDE) over Stochastic Differential Equation (SDE) through asset value changes.

This paper is arranged as follows: Section 2.1 is Materials and method, Section 3.1 presents results and discussion while the paper is concluded in Section 4.1.

2.1 Materials and Method

In the derivation of the stochastic model representing asset value with or without delay, some theorems and definitions are considered. They are as follows:

Definition 1: Stochastic process is seen as a statistical event that evolves time in accordance to probabilistic laws.

Definition 2: Mathematically, a stochastic process may be defined as a collection of random variables which are ordered in time and defines at a set of time points which may be continuous or discrete.

Definition 3: A stochastic process whose finite dimensional probability distributions are all Gaussian, which is (Normal distribution).

Definition 4: A Stochastic Differential Equation is a differential equation with stochastic term. Therefore, assume that $(\Omega, F, \wp)$ is a probability space with filtration $\{f_t\}_t \geq 0$ and $W(t) = (W_1, W_1, \ldots, W_m)^T, t \geq 0$ an m-dimensional Brownian motion on the given probability space. We have SDE in coefficient functions of f and g as follows:

$$dX = f(t, X)\,dt + g(t, X)\,dZ, \quad 0 \leq t \leq T \tag{1}$$

$$X(0) = x_0 \tag{2}$$

where $T > 0$, x_0 is an n-dimensional random variable and coefficient functions are in the form $f:[0,T]\times R^n$ and $g:[0,T]\times R^n \rightarrow R^{m\times n}$. SDE can also be written in the form of integral as follows:

$$X(t) = x_0 + \int_0^t f(S, X(S))\,dS + \int_0^t g(S, X(S))\,dZ(S) \tag{3}$$

where dX and dZ are known as stochastic differentials. The $\mathbb{R}^n$ is a valued stochastic process $X(t)$ see, George and Kenneth (2017).

Theorem 1.1: let $T > 0$, be a given final time and assume that the coefficient functions $f:[0,\mathrm{T}]\times\mathbb{R}^n \rightarrow \mathbb{R}^n$ *and* $g:[0,T]\times\mathbb{R}^n \rightarrow \times\mathbb{R}^{m\times n}$ are continuous. Moreover, $\exists$ finite constant numbers λ *and* β such that $\forall t \in [0,\mathrm{T}]$ and for all $x, y \in \mathbb{R}^n$, the drift and diffusion term satisfy

$$\|f(t,x) - f(t,y)\| + \|g(t,x) - g(t,y)\| \leq \lambda\|x - y\|$$

$$\|f(t,x)\| + \|g(t,x)\| - \|g(t,x)\| \leq \beta(1 + \|x\|)$$

Suppose also that x_0 is any $\mathbb{R}^n$ -valued random variable such that $E(\|x_0\|^2) < \infty$ then the above SDE has a unique solution X in the interval $[0,T]$. Moreover, it satisfies $E\left(\sup_{0\leq t\leq T}\|X(t)\|^2\right) < \infty$ .the proof of the theorem 1.1 is seen in Lambert and Lapeyre (2007)..

Theorem 1.2: (Ito's lemma): Let $f(S,t)$ be a twice continuous differential function on $[0,\infty)\times A$ and let S_t denotes an Ito's process

$$dS_t = a_t dt + b_t dz(t), t > 0$$

Applying Taylor series expansion of F gives:

$$F_t = F(S_t, t)$$

$$dF_t = \frac{\partial F}{\partial S_t} dS_t + \frac{\partial F}{\partial t} dt + \frac{1}{2}\frac{\partial^2 F}{\partial S_t^2}(dS_t)^2 + ..$$

Ignoring the higher order in equations we have:

$$dF_t = \frac{\partial F}{\partial S_t}\left(a_t dt + b_t dz(t)\right) + \frac{\partial F}{\partial t} dt + \frac{1}{2}\frac{\partial^2 F}{\partial S_t^2}\left(a_t dt + b_t dz(t)\right)^2$$

$$dF_t = \frac{\partial F}{\partial S_t}\left(a_t dt + b_t dz(t)\right) + \frac{\partial F}{\partial t} dt + \frac{1}{2}\frac{\partial^2 F}{\partial S_t^2} b_t^2 dt$$

$$dF_t = \left(\frac{\partial F}{\partial S_t} a_t + \frac{\partial F}{\partial t} + \frac{1}{2}\frac{\partial^2 F}{\partial S_t^2} b_t^2\right) dt + \frac{\partial F}{\partial S_t} b_t dz(t)$$

More so, given the variable $S(t)$ denotes stock price, then following GBM implies the function $F(S,t)$, then the Ito's lemma gives:

$$dF = \left(\mu S\frac{\partial F}{\partial S} + \frac{\partial F}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 F}{\partial S^2}\right) dt + \sigma S \frac{\partial F}{\partial S} dz(t)$$

2.1.1 Formulation of the Problem

We consider stochastic differential equations imposed with the dynamics of return rates which is said to have a complete probability space $(\Omega, F, \wp)$ such that a finite time investment horizon $T > 0$. Time delay is paramount in assessing asset value so adding delay term in a model gives more realistic and better model. Therefore, delay term is added to the non-Brownian term of the stochastic differential equation, Following Amadi and Okpoye (2022), Amadi and Charles (2022), the rate of returns gives as
we obtained the following system of delay stochastic differential equations:

$$dX_2 = (t-\tau)(\theta_1 + \theta_2)^2 X_2(t) dt + \sigma X_2(t) dZ^2(t) \tag{4}$$

$$dX_4 = (\theta_1 + \theta_2)^2 X_4(t) dt + \sigma X_4(t) dZ^4(t) \tag{5}$$

The initial conditions are:

$$X_1(0) = X_0, X_3(0) = X_0 \tag{6}$$

where, $X_3(t)$ *and* $X_4(t)$ are asset prices, The expression dZ, which contains the randomness that is certainly a characteristic of asset prices is called a Wiener process or Brownian motion, λ_1 and λ_2 represents rate of returns of first and second investments respectively, $\lambda_1 + \lambda_2$ is the additive effects.

2.2 Method of Solution

Analytical Solution of Additive Effect of the rate of returns on Investment with Delay

we let $f(X_3, t) = \ln X_3$ so that upon differentiation, we have:

$$\frac{\partial f}{\partial X_3} = \frac{1}{X_3} \tag{7}$$

$$\frac{\partial^2 f}{\partial X_3^2} = -\frac{1}{X_3^2} \tag{8}$$

$$\frac{\partial f}{\partial t} = 0 \tag{9}$$

Thus; from Ito's lemma, we have

$$df(X_3, t) = \frac{\partial f}{\partial X_3} dX_3 + \frac{1}{2}\frac{\partial^2 f}{\partial X_3^2}(dX_3)^2 + \frac{\partial f}{\partial t} dt \tag{10}$$

$$df(X_3, t) = \sigma X_3 \frac{\partial f}{\partial X_3} dZ + \left((t-\tau)(\theta_1+\theta_2)^2 X_3 \frac{\partial f}{\partial X_3} dt\right) + \frac{1}{2}\frac{\partial^2 f}{\partial X_3^2}\left((t-\tau)(\theta_1+\theta_2)^2 X_3 dt + \sigma X_3 dZ\right)^2 + \frac{\partial f}{\partial t} dt \tag{11}$$

$$df(X_3, t) = \sigma X_3 \frac{\partial f}{\partial X_3} dZ + \left((t-\tau)(\theta_1+\theta_2)^2 X_3 \frac{\partial f}{\partial X_3} dt\right)$$

$$+\left((t-\tau)(\theta_1+\theta_2)^2 X_3^2 \frac{1}{2}\frac{\partial^2 f}{\partial X_3^2}(dt)^2 + 2\sigma(t-\tau)(\theta_1+\theta_2)^2 X_3^2 \frac{1}{2}\frac{\partial^2 f}{\partial X_3^2} dtdZ + \sigma^2 X_1^2 \frac{1}{2}\frac{\partial^2 f}{\partial X_3^2}(dZ)^2\right) + \frac{\partial f}{\partial t} dt \tag{12}$$

Since $dt \to 0, (dt)^2 \to 0, \text{ and } (dZ)^2 \to dt$

$$df(X_3, t) = \sigma X_3 \frac{\partial f}{\partial X_3} dZ + (t-\tau)(\theta_1+\theta_2)^2 X_3 \frac{\partial f}{\partial X_3} dt + \sigma^2 X_3^2 \frac{1}{2}\frac{\partial^2 f}{\partial X_3^2} dt + \frac{\partial f}{\partial t} dt \tag{13}$$

Simplifying equation (3.30), we have

$$df(X_3, t) = \sigma X_3 \frac{\partial f}{\partial X_3} dZ + \left((t-\tau)(\theta_1+\theta_2)^2 X_3 \frac{\partial f}{\partial X_3} + \frac{1}{2}\sigma^2 X_3^2 \frac{\partial^2 f}{\partial X_3^2} + \frac{\partial f}{\partial t}\right) dt \tag{14}$$

Substituting equations , we have:

$$df(X_3, t) = \sigma \frac{X_3}{X_3} dZ + \left((t-\tau)(\theta_1+\theta_2)^2 \frac{X_3}{X_3} - \frac{\sigma^2 X_3^2}{2X_3^2} + \frac{\partial f}{\partial t}\right) dt \tag{15}$$

Simplifying equation (3.32), we have:

$$df(X_3, t) = \sigma dZ + \left((t-\tau)(\theta_1+\theta_2)^2 - \frac{\sigma^2}{2}\right) dt \tag{16}$$

We know that, $f(X_3, t) = \ln X_3$ (17)

Substituting equation we have:

$$d(\ln X_3) = \sigma dZ + \left((t-\tau)(\theta_1\theta_2)^2 - \frac{\sigma^2}{2}\right) dt \tag{18}$$

Integrating equation we have:

$$\int_0^t d(\ln X_3) = \int\left(\left(t-\tau\right)\left(\theta_1\theta_2\right)^2 - \frac{\sigma^2}{2}\right)dt + \sigma\int dZ^1 \qquad (19)$$

$$\ln X_3 - \ln X_0 = \frac{\left(t-\tau\right)^2}{2}\left(\theta_1+\theta_2\right)^2 - \frac{\sigma^2 t}{2} + \sigma Z \qquad (20)$$

$$\ln\left(\frac{X_3}{X_0}\right) = \frac{\left(t-\tau\right)^2}{2}\left(\theta_1+\theta_2\right)^2 - \frac{\sigma^2 t}{2} + \sigma Z \qquad (21)$$

$$\frac{X_3}{X_0} = \exp\left(\frac{\left(t-\tau\right)^2}{2}\left(\theta_1+\theta_2\right)^2 - \frac{\sigma^2 t}{2} + \sigma Z\right) \qquad (22)$$

$$X_3(t) = X_0\exp\left(\frac{\left(t-\tau\right)^2}{2}\left(\theta_1+\theta_2\right)^2 - \frac{\sigma^2 t}{2} + \sigma Z\right) \qquad (23)$$

Analytical Solution of Additive Effect of the rate of returns on Investment without Delay
It can be written as:

$$f(X_4, t) = \ln X_4 \qquad (24)$$

$$\frac{\partial f}{\partial X_4} = \frac{1}{X_4} \qquad (25)$$

$$\frac{\partial^2 f}{\partial X_4^2} = -\frac{1}{X_4^2} \qquad (26)$$

$$\frac{\partial f}{\partial t} = 0 \qquad (27)$$

Thus; from Ito's lemma, we have

$$df(X_4, t) = \frac{\partial f}{\partial X_4}dX_4 + \frac{1}{2}\frac{\partial^2 f}{\partial X_4^2}\left(dX_4\right)^2 + \frac{\partial f}{\partial t}dt \qquad (28)$$

$$df(X_4, t) = \sigma X_4\frac{\partial f}{\partial X_4}dZ^2 + \left(\left(\theta_1+\theta_2\right)^2 X_4\frac{\partial f}{\partial X_4}dt\right) + \frac{1}{2}\frac{\partial^2 f}{\partial X_4^2}\left(\left(\theta_1+\theta_2\right)^2 X_4 dt + \sigma X_4 dZ^2\right)^2 + \frac{\partial f}{\partial t}dt \qquad (29)$$

$$df(X_4, t) = \sigma X_4\frac{\partial f}{\partial X_4}dZ^2 + \left(\left(\theta_1+\theta_2\right)^2 X_4\frac{\partial f}{\partial X_4}dt\right)$$
$$+\left(\left(\theta_1+\theta_2\right)^2 X_4^2\frac{1}{2}\frac{\partial^2 f}{\partial X_4^2}\left(dt\right)^2 + 2\sigma\left(\theta_1+\theta_2\right)^2 X_4^2\frac{1}{2}\frac{\partial^2 f}{\partial X_4^2}dtdZ + \sigma^2 X_4^2\frac{1}{2}\frac{\partial^2 f}{\partial X_4^2}\left(dZ\right)^2\right) + \frac{\partial f}{\partial t}dt \qquad (30)$$

Since $dt \to 0, \left(dt\right)^2 \to 0,$ and $\left(dZ\right)^2 \to dt$

$$df(X_4, t) = \sigma X_4\frac{\partial f}{\partial X_4}dZ^2 + \left(\theta_1+\theta_2\right)^2 X_4\frac{\partial f}{\partial X_4}dt + \sigma^2 X_4^2\frac{1}{2}\frac{\partial^2 f}{\partial X_4^2}dt + \frac{\partial f}{\partial t}dt \qquad (31)$$

Simplifying we have

$$df(X_4,t)=\sigma X_4\frac{\partial f}{\partial X_4}dZ+\left((\theta_1+\theta_2)^2 X_4\frac{\partial f}{\partial X_4}+\frac{1}{2}\sigma^2X_4^2\frac{\partial^2 f}{\partial X_4^2}+\frac{\partial f}{\partial t}\right)dt \tag{32}$$

Substituting we have:

$$df(X_4,t)=\sigma\frac{X_4}{X_4}dZ+\left((\theta_1+\theta_2)^2\frac{X_4}{X_4}-\frac{\sigma^2X_4^2}{2X_4^2}+\frac{\partial f}{\partial t}\right)dt \tag{32}$$

Simplifying equation (3.49), we have:

$$df(X_4,t)=\sigma dZ+\left((\theta_1+\theta_2)^2-\frac{\sigma^2}{2}\right)dt \tag{33}$$

We recall equation (3.41a), $f(X_4,t)=\ln X_4$ (34)

Substituting equation (3.51) into equation (3.50), we have:

$$d(\ln X_4)=\sigma dZ+\left((\theta_1+\theta_2)^2-\frac{\sigma^2}{2}\right)dt \tag{35}$$

Integrating equation (5), we have:

$$\int_0^t d(\ln X_4)=\int\left((\theta_1+\theta_2)^2-\frac{\sigma^2}{2}\right)dt+\sigma\int dZ \tag{36}$$

$$\ln X_4-\ln X_0=\left(\frac{(\theta_1+\theta_2)^2}{2}-\frac{\sigma^2}{2}\right)t+\sigma Z \tag{37}$$

$$\ln\left(\frac{X_4}{X_0}\right)=\left(\frac{(\theta_1+\theta_2)^2}{2}-\frac{\sigma^2}{2}\right)t+\sigma Z \tag{38}$$

$$\frac{X_4}{X_0}=\exp\left(\left(\frac{(\theta_1+\theta_2)^2}{2}-\frac{\sigma^2}{2}\right)t+\sigma Z\right) \tag{39}$$

$$X_4(t)=X_0\exp\left(\left(\frac{(\theta_1+\theta_2)^2}{2}-\frac{\sigma^2}{2}\right)t+\sigma Z\right) \tag{40}$$

3. 1 Results and Discussion

This Section presented numerical results whose solutions are in (3.14) - (3.17). We have the following:

Table 1: The impact of time delay in the assessment of Asset values with additive effects through the solution below:

$$X_2(t)=X_0\exp\left\{(t-\tau)\left((\theta_1+\theta_2)^2-\frac{1}{2}\sigma^2\right)t+\sigma dz(t)\right\}\quad t=2,\ \ dz=1$$

τ	S_0	σ	$(\theta_1+\theta_2)$	$X_2(t)$	$(\theta_1+\theta_2)$	$X_2(t)$

	5.00	0.2	1.0000	194.3067	2.0000	7056346.005
0.25	6.00	0.2	1.0000	233.1681	2.0000	8467615.206
	5.70	0.2	1.0000	221.5097	2.0000	8044234.446
	5.20	0.2	1.0000	165.4483	2.0000	3297445.466
0.35	4.60	0.2	1.0000	146.3581	2.0000	2916790.99
	4.00	0.2	1.0000	127.2679	2.0000	2536496.513
	3.70	0.2	1.0000	96.3833	2.0000	1054242.249
0.45	3.90	0.2	1.0000	101.5932	2.0000	1111228.317
	3.50	0.2	1.0000	91.1734	2.0000	997256.1817
	3.31	0.2	1.0000	70.5942	2.0000	423770.8709
0.55	3.75	0.2	1.0000	79.9783	2.0000	480102.9504
	4.00	0.2	1.0000	85.3102	2.0000	512109.8138

Results in Table 1 extended the analysis to additive return structures and confirmed that the delay parameter similarly produces significant increases in asset values under additive effects. As with the multiplicative case, this result reflects the practical reality that investors who defer the disposal of assets — whether physical properties, stocks, or business interests — ultimately earn more from those assets than those who liquidate immediately.

This finding directly agrees with the work of Okpoye et al. (2023), who studied additive effects alongside multiplicative effects in their empirical asset value framework and found that the additive inverse effects provided the highest rate of returns throughout the trading days. The present study extends this result by demonstrating that when an additive return structure is embedded within an SDDE framework, the delay parameter further amplifies asset value growth beyond what static additive SDE models can produce.

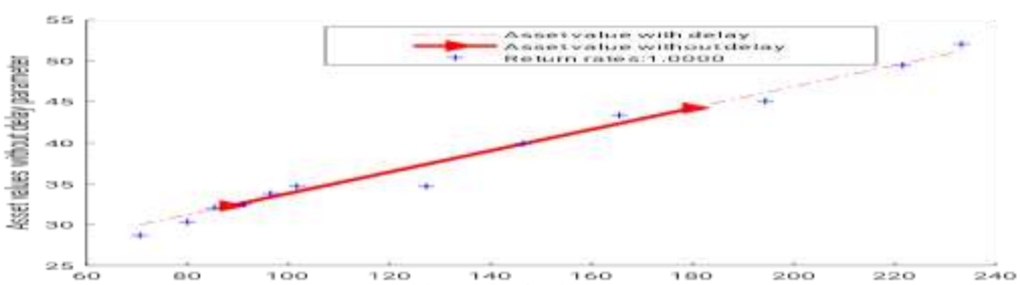

Figure 1: Graphical representation of two asset values (with and without delay) parameter which follows additive effects with:1.0000.

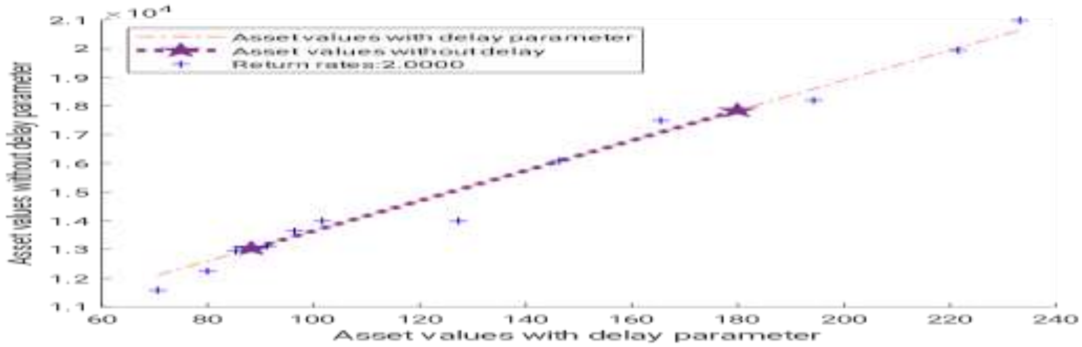

Figure 2: Graphical representation of two asset values (with and without delay) parameter which follows additive effects with:1.0000.

The quantile-quantile (Q-Q) plots in Figures 1 and 2 confirmed that the two asset value populations — with and without delay — do not share a common distribution under the additive structure. This result is consistent with the findings of Zhou et al. (2016), further reinforcing that delay introduces a structural divergence in asset value distributions regardless of whether the return structure is multiplicative or additive.

Table 2: The assessment of Asset values with additive effects through the solution below:

$$X_4(t) = X_0 \exp\left\{\left((\theta_1+\theta_2)^2 - \frac{1}{2}\sigma^2\right)t + \sigma\, dz(t)\right\},\ dz = 1 \text{ and } t = 2$$

S_0	σ	$(\theta_1+\theta_2)$	$X_4(t)$	$(\theta_1+\theta_2)$	$X_4(t)$
5.00	0.2	1.0000	43.3557	2.0000	17490.9330
6.00	0.2	1.0000	52.0268	2.0000	20989.1196
5.70	0.2	1.0000	49.4255	2.0000	19939.6636
5.20	0.2	1.0000	45.0899	2.0000	18190.5703
4.60	0.2	1.0000	39.8872	2.0000	16091.6584
4.00	0.2	1.0000	34.6846	2.0000	13992.7464
3.70	0.2	1.0000	32.0832	2.0000	12943.2904
3.90	0.2	1.0000	33.8174	2.0000	13642.9278
3.50	0.2	1.0000	30.3490	2.0000	12243.6531

3.31	0.2	1.0000	28.7015	2.0000	11578.9977
3.75	0.2	1.0000	32.5168	2.0000	13118.1998
4.00	0.2	1.0000	34.6846	2.0000	13992.7464

Table 2, which displays asset values without delay under additive effects, showed comparatively low returns, confirming that immediate liquidation of assets yields suboptimal financial outcomes. This agrees with Amadi and Okpoye (2022), who found that investment structures that do not account for time-varying effects produce limited returns, and with Amadi and Wobo (2022), whose mathematical model for estimating stock market price changes demonstrated that short-run and long-run stock behaviour differ substantially — a difference that the present study attributes, in part, to the presence or absence of delay mechanisms.

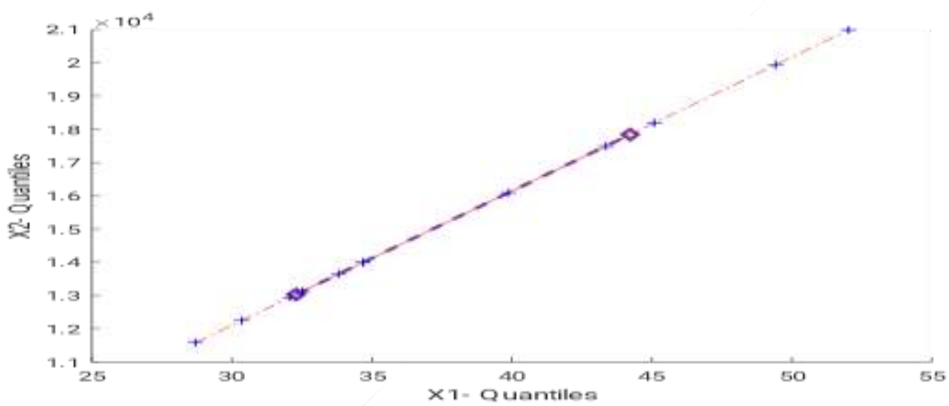

Figure 3: Graphical representation of two asset values (with and without delay) parameter which follows additive effects.

H_0 : The asset value functions $X_2(t)$ and $X_4(t)$ comes from a common distribution

H_1 : They are not from a common distribution

Results indicates that the alternative hypothesis of KS was accepted because it is the hypothesis we are will to accept if we reject the null hypothesis hence the p-values are not significant.

Taken together, the additive effect findings confirm and extend the prior literature: delay is a structurally significant variable that consistently enhances asset value accumulation across both multiplicative and additive return frameworks. The present study's contribution lies in formally quantifying this enhancement through an SDDE framework and statistically validating it through the KS test a level of rigour not previously achieved in the reviewed literature on Nigerian capital market stochastic modelling.

4.1 Conclusion

The SDDE model offers more realistic asset value estimations than the standard SDE because it accounts for memory effects and market inertia. Higher initial stock prices amplify the trajectory of asset value growth over time.Variability in asset values (as reflected by standard deviation) directly corresponds to volatility levels, implying that high-value assets exhibit larger fluctuations.The Kolmogorov-Smirnov (KS) test confirmed that asset values, with and without delay, do not originate from a common distribution, underscoring the distinct dynamics captured by the delay-inclusive model.Overall, this research underscores that the inclusion of stochastic delay structures provides a more comprehensive understanding of real-world market behavior, offering improved predictive capability for investors and analysts in the oil and gas sector. The study recommends the adoption of both Stochastic Differential Equation (SDE) and Stochastic Delay Differential Equation (SDDE) in the estimation of asset values.

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